4. Expanding along the first row:

$$\begin{vmatrix} 1 & 3 & 5 \\ 2 & 1 & 1 \\ 3 & 4 & 2 \end{vmatrix} = 1\begin{vmatrix} 1 & 1 \\ 4 & 2 \end{vmatrix} - 3\begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} + 5\begin{vmatrix} 2 & 1 \\ 3 & 4 \end{vmatrix} = 1(-2) - 3(1) + 5(5) = 20$$

Expanding along the second column:

$$\begin{vmatrix} 1 & 3 & 5 \\ 2 & 1 & 1 \\ 3 & 4 & 2 \end{vmatrix} = (-1)^{1+2} \cdot 3\begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} + (-1)^{2+2} \cdot 1\begin{vmatrix} 1 & 5 \\ 3 & 2 \end{vmatrix} + (-1)^{3+2} \cdot 4\begin{vmatrix} 1 & 5 \\ 2 & 1 \end{vmatrix} = -3(1) + 1(-13) - 4(-9) = 20$$

8. Expanding along the first row:

$$\begin{vmatrix} 8 & 1 & 6 \\ 4 & 0 & 3 \\ 3 & -2 & 5 \end{vmatrix} = 8\begin{vmatrix} 0 & 3 \\ -2 & 5 \end{vmatrix} - 1\begin{vmatrix} 4 & 3 \\ 3 & 5 \end{vmatrix} + 6\begin{vmatrix} 4 & 0 \\ 3 & -2 \end{vmatrix} = 8(6) - 1(11) + 6(-8) = -11$$

$$-9 + 0 + 40 - 0 - 30 - 0 = 1$$

16. $\begin{vmatrix} 0 & 5 & 1 \\ 4 & -3 & 0 \\ 2 & 4 & 1 \end{vmatrix} = (0)(-3)(1) + (5)(0)(2) + (1)(4)(4) - (2)(-3)(1) - (4)(0)(0) - (1)(4)(5) =$

$$0 + 0 + 16 - (-6) - 0 - 20 = 2$$

24. $$\begin{vmatrix} a & b & c \\ 3 & 2 & 2 \\ 6 & 5 & 6 \end{vmatrix} = a(2) - b(6) + c(3) = 2a - 6b + 3c,$$

$$\begin{vmatrix} 3 & 2 & 2 \\ a & b & c \\ 6 & 5 & 6 \end{vmatrix} = 3(6b - 5c) - 2(6a - 6c) + 2(5a - 6b) = -2a + 6b - 3c$$

The row operation swaps rows 1 and 2 of the matrix, and the sign of the determinant is reversed.

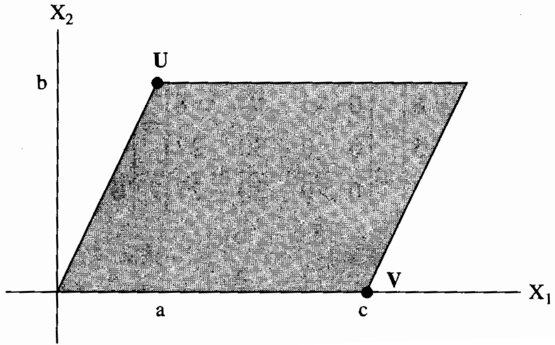
$$\dots = (\det E)(\det A)$$

34. $E = \begin{bmatrix} 1 & 0 \\ 0 & k \end{bmatrix}$, $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $EA = \begin{bmatrix} a & b \\ kc & kd \end{bmatrix}$

$\det E = k$, $\det A = ad - bc$,

$\det EA = a(kd) - (kc)b = k(ad - bc) = (\det E)(\det A)$

42. The area of the parallelogram determined by $\mathbf{u} = \begin{bmatrix} a \\ b \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} c \\ 0 \end{bmatrix}$, $\mathbf{u} + \mathbf{v}$, and $\mathbf{0}$ is cb, since the base of the parallelogram has length c and the height of the parallelogram is b.



Also note that $\det\begin{bmatrix}\mathbf{u} & \mathbf{v}\end{bmatrix} = \det\begin{bmatrix} a & c \\ b & 0 \end{bmatrix} = -cb$, and $\det\begin{bmatrix}\mathbf{v} & \mathbf{u}\end{bmatrix} = \det\begin{bmatrix} c & a \\ 0 & b \end{bmatrix} = cb$. The determinant of the matrix whose columns are those vectors which define the sides of the parallelogram adjacent to **0** either is equal to the area of the parallelogram or is equal to the negative of the area of the parallelogram.